

Dipion invariant mass spectrum in $X(3872) \rightarrow J/\psi \pi \pi$

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Abstract

It is pointed out that dipion invariant mass spectrum in $X(3872) \rightarrow \pi \pi J/\psi$ is a useful probe for the J^{PC} quantum number of the $X(3872)$, complementary to the angular distributions. If $X(3872)$ is a 1P_1 state, the dipion has a peak at low $m_{\pi\pi}$ region, which is not in accord with the preliminary Belle data. If $X(3872)$ is a 3D_2 or 3D_3 state, the dipion spectrum shows a peak at high $m_{\pi\pi}$ region, which is broader than the ρ resonance that might come from the decay of a molecular state: $(D\overline{D}^*)(I=1) \rightarrow J/\psi \rho \rightarrow J/\psi \pi^+ \pi^-$. Better measurement of $m_{\pi\pi}$ spectrum will shed light on the nature of $X(3872)$.

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I. INTRODUCTION

B decay is a nice place to look for charmonium states which are either above the $D^{(*)}\bar{D}^{(*)}$ threshold or have quantum numbers that are not accessible from n^3S_1 states by cascade decays. $2^3P_J, 3^3D_J$ and 1^1P_1 states are such examples [1, 2, 3]. In this regard, the advance in NRQCD enabled us to estimate the inclusive B decays into charmonia without infrared divergence problem [4].

Recently, Belle collaboration reported a new narrow resonance $X(3872)$ in

$$B \rightarrow XK \rightarrow (J/\psi\pi\pi)K$$

decay channel [5], which was subsequently confirmed by CDF Collaboration [6]. It is important to identify the nature of this new narrow state, and there are several works on this state already [7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19]. In particular, Pakvasa and Suzuki pinned down possible quantum numbers for $J^{PC}(X)$ [7] as follows:

- If it is a charmonium, it should be either $1^3D_2(2^{--})$ or $2^1P_1(1^{+-})$ [27]. Also $1^3D_3(3^{--})$ is another possibility [20].
- If $X(3872)$ is the $D\bar{D}^*$ molecular state, it should be either $J^{PC} = 1^{+-}$ with $I = 0$, or $J^{PC} = 1^{++}$ with $I = 1$.

Also they pointed out that the dipion angular spectrum could be useful to determine the J^{PC} quantum number of $X(3872)$ [7].

In this letter, we show that the dipion invariant mass spectrum in $X \rightarrow J/\psi\pi\pi$ provides independent information on the nature of $X(3872)$. Our method is complementary to the angular correlations suggested in Ref. [7], and is already useful to eliminate a possibility that $X = 1^1P_1$ state.

II. CHIRAL LAGRANGIAN INVOLVING HEAVY QUARKONIA AND LIGHT VECTOR MESONS

Hadronic transition between heavy quarkonia can be described in terms of QCD multipole expansion or chiral perturbation theory. Since the allowed dipion invariant mass in $X(k, \epsilon_X) \rightarrow J/\psi(k', \epsilon_\psi) \pi(p_1)\pi(p_2)$ is

$$2m_\pi \leq m_{\pi\pi} \leq (M_X - M_{J/\psi}) = 775 \text{ GeV} \approx m_\rho,$$

both approaches will be suitable to our purpose. In this letter, we use chiral lagrangian approach, since we are interested only in the spectrum and not in the absolute decay rate. Under global chiral $SU(3)_L \times SU(3)_R$ transformation, the pion field $\Sigma(x) \equiv \exp(2i\pi(x)/f_\pi)$ transforms as

$$\Sigma(x) \rightarrow L\Sigma(x)R^\dagger. \quad (1)$$

Under parity P and charge conjugation C , the pion fields transform as

$$\begin{aligned} P : \pi(t, \vec{x}) &\rightarrow -\pi(t, -\vec{x}) \\ \Sigma(t, \vec{x}) &\rightarrow \Sigma^\dagger(t, -\vec{x}) \\ C : \pi(t, \vec{x}) &\rightarrow \pi(t, \vec{x})^T \\ \Sigma(t, \vec{x}) &\rightarrow \Sigma(t, \vec{x})^T \end{aligned} \quad (2)$$

In order to introduce other matters such as ρ or $X(3872)$ etc., it is convenient to define another field $\xi(x)$ by $\Sigma(x) \equiv \xi^2(x)$, which transforms as

$$\xi(x) \rightarrow L\xi(x)U^\dagger(x) = U(x)\xi(x)R^\dagger. \quad (3)$$

The 3×3 matrix field $U(x)$ depends on Goldstone fields $\pi(x)$ as well as the SU(3) transformation matrices L and R . It is convenient to define two vector fields with following properties under chiral transformations:

$$\begin{aligned} V_\mu &= \frac{1}{2}(\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger), & V_\mu &\rightarrow UV_\mu U^\dagger + U \partial_\mu U^\dagger, \\ A_\mu &= \frac{i}{2}(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger), & A_\mu &\rightarrow UA_\mu U^\dagger. \end{aligned} \quad (4)$$

Note that V_μ transforms like a gauge field.

Chiral symmetry is explicitly broken by non-vanishing current-quark masses and electromagnetic interactions. The former can be included by regarding the quark-mass matrix $m = \text{diag}(m_u, m_d, m_s)$ as a spurion with transformation property $m \rightarrow LmR^\dagger = RmL^\dagger$. It is more convenient to use $\xi m \xi + \xi^\dagger m \xi^\dagger$, which transforms as an SU(3) octet:

$$(\xi m \xi + \xi^\dagger m \xi^\dagger) \rightarrow U(x)(\xi m \xi + \xi^\dagger m \xi^\dagger)U^\dagger(x) \quad (5)$$

One can introduce light vector mesons ρ_μ , which transforms as

$$\rho_\mu(x) \rightarrow U(x)\rho_\mu(x)U^\dagger(x) + U(x)\partial_\mu U^\dagger(x), \quad (6)$$

under global chiral transformations [21]. Then $\rho_\mu(x)$ transforms as a gauge field under local SU(3)'s defined by Eq. (1), as V_μ does. The covariant derivative \mathcal{D}_μ can be defined using ρ_μ instead of V_μ . Note that $(\rho_\mu - V_\mu)$ has a simple transformation property under chiral transformation:

$$(\rho_\mu - V_\mu) \rightarrow U(x)(\rho_\mu - V_\mu)U^\dagger(x), \quad (7)$$

Both ρ_μ and V_μ are C - and P -odd. Then it is straightforward to construct chiral invariant lagrangian using ρ fields. In terms of a field strength tensor $\rho_{\mu\nu}$,

$$\mathcal{L}_\rho = -\frac{1}{2} \text{Tr}(\rho_{\mu\nu}\rho^{\mu\nu}) + \frac{1}{2} m_\rho^2 \text{Tr}(\rho_\mu - V_\mu)^2 \quad (8)$$

The 1st term is the kinetic term, and the second term is the ρ mass term.

Since the final charmonium is moving very slowly in the rest frame of the initial state $X(3872)$, we can use the heavy particle effective theory approach, by introducing a velocity dependent field $X_v(x) \equiv X e^{im_X v \cdot x}$, and similarly for J/ψ field $\psi_v(x)$ [22]. If X_v is an isosinglet, it is chially invariant. If X_v is an isovector $I(X_v) = 1$, then it transforms as

$$X_v(x) \rightarrow U(x)X_v(x)U^\dagger(x). \quad (9)$$

This will be useful when we consider the case $J^{PC}(X) = 1^{++}, I = 1$ in Sec. VI.

Then we can construct chiral lagrangian which is invariant under parity P and charge conjugation C , using the pion field $U(x)$, $X_v(x)$, a chiral singlet $\psi_v(x)$, the metric tensor $g_{\mu\nu}$ and the Levi-Civita tensor $\epsilon_{\mu\nu\alpha\beta}$. Transformation properties of $X_v(x)$, $\psi_v(x)$ and v^μ under parity and charge conjugation are given in Table 1.

Using our chiral lagrangian, we write down the amplitude for $X \rightarrow J/\psi \pi \pi$ and derive the dipion mass spectrum. This approach gives the same results as QCD multipole expansion for

TABLE I: Transformation properties of X_v , ψ_v and v under parity and charge conjugation.

Fields	P	C
v^μ	$v_\mu = (v^0, -\vec{v})$	$v^\mu = (v^0, \vec{v})$
ψ_v^μ	$\psi_{v\mu}$	$-\psi_v^\mu$
$X_v^{\mu\nu}({}^3D_2)$	$-X_{v\mu\nu}$	$-X_v^{\mu\nu}$
$X_v^{\mu\alpha\beta}({}^3D_3)$	$+X_{v\mu\alpha\beta}$	$-X_v^{\mu\alpha\beta}$
$X_v^\mu({}^1P_1)$	$-X_{v\mu}$	$-X_v^\mu$
$X_v^\mu(J^{PC} = 1^{++})$	$-X_{v\mu}$	X_v^μ

the Lorentz and chiral structures of the amplitude, upto the unknown overall normalization. Similar analyses for $\Upsilon(3S) \rightarrow \Upsilon(1S)\pi\pi$ were presented in Ref.s [23].

A remark is in order before we proceed. For $\psi' \rightarrow J/\psi\pi\pi$ or $\Upsilon(3S) \rightarrow \Upsilon(1S)\pi\pi$, there are 4 independent chirally invariant operators to lowest order in pion momenta:

$$\mathcal{M} \sim \epsilon \cdot \epsilon' (q^2 + BE_1 E_2 + Cm_\pi^2) + D \left(\epsilon \cdot p_1 \epsilon' \cdot p_2 + \epsilon \cdot p_2 \epsilon' \cdot p_1 \right), \quad (10)$$

with $B, C, D \sim O(1)$, but QCD multipole expansion predicts that $B = D = 0$. B, C, D being apriori unknown, one cannot make definite predictions for the $m_{\pi\pi}$ spectrum. Rather one has to fix B, C, D from the $m_{\pi\pi}$ spectrum as done in Ref.s [23]. On the other hand, there is only single chirally invariant operator to lowest order in pion momenta in the three options for the $J^{PC}(X)$ we consider in this work, and we can make a definite prediction for $m_{\pi\pi}$ spectrum, unlike the case for $\psi' \rightarrow J/\psi\pi\pi$ or $\Upsilon(3S) \rightarrow \Upsilon(1S)\pi\pi$.

III. $1^3D_2(2^{--}) \rightarrow J/\psi\pi\pi$

This decay is described by the following chiral lagrangian with heavy quarkonia:

$$\mathcal{L} = g({}^3D_2) \epsilon^{\mu\nu\alpha\beta} v_\mu \psi_{v,\nu} X_{v,\alpha\rho} \text{Tr} [\partial_\beta \Sigma \partial^\rho \Sigma^\dagger] + h.c. \quad (11)$$

where the coupling $g({}^3D_2)$ is unknown parameter, that should be determined by the data or could be calculated within QCD multipole expansion. (See Ref. [24] for an explicit form of $g({}^3D_2)$.) Then the amplitude for this decay is given by

$$\mathcal{M} \sim \epsilon_{ijk} \epsilon_\psi^i \epsilon_X^{jl} (p^k p^l - q^k q^l), \quad (12)$$

where $p \equiv p_1 - p_2$ and $q \equiv p_1 + p_2$, and $i, j, k, l = 1, 2, 3$ are space indices. Note that there is only one operator that contributes to the decay $1^3D_2(2^{--}) \rightarrow J/\psi\pi\pi$, so that we can predict the $\pi\pi$ spectrum without any ambiguity. The overall normalization is determined by the decay rate, but is irrelevant to our discussion for the $\pi\pi$ spectrum. Now it is straightforward to calculate the dipion invariant mass spectrum using the above amplitude. The resulting spectrum is shown in Fig. 1 in the (red) solid curve. Note that the dipion invariant mass has a peak at high $m_{\pi\pi}$ region, which is consistent with the preliminary Belle data [5].

IV. ${}^3D_3(3^{--}) \rightarrow J/\psi\pi\pi$

Another possibility for $X(3872)$ is that it is a ${}^3D_3(3^{--})$ state, whose transformation properties under parity and charge conjugation is listed in Table 1. In this case again, there is a single chirally invariant operator that contributes to $X \rightarrow J/\psi\pi\pi$:

$$\mathcal{L} = g({}^3D_3) \psi_{v\mu} X_{v\alpha\beta}^\mu \text{Tr} [\partial^\alpha \Sigma \partial^\beta \Sigma^\dagger], \quad (13)$$

which leads to the following amplitude:

$$\mathcal{M} \sim \epsilon_\mu \epsilon_X^{\mu\alpha\beta} (p_{1\alpha} p_{2\beta} + p_{2\alpha} p_{1\beta}). \quad (14)$$

Then we can predict the $m_{\pi\pi}$ spectrum without ambiguity as before. The spin averaged squared amplitude for ${}^3D_3 \rightarrow J/\psi\pi\pi$ is equal to that for ${}^3D_2 \rightarrow J/\psi\pi\pi$, which was also discussed within QCD multipole expansion in Ref. [24]. The resulting $m_{\pi\pi}$ spectra are the same as shown in the (red) solid curve in Fig. 1.

V. $X(1^{+-}, I=0) \rightarrow J/\psi\pi\pi$

If X is a charmonium 1P_1 state with $J^{PC} = 1^{+-}$ and $I=0$, the relevant chiral lagrangian is given by

$$\mathcal{L} = g({}^1P_1) \epsilon^{\mu\nu\alpha\beta} X_{v\mu} \psi_{v\nu} \text{Tr} [\partial_\alpha \Sigma \partial_\beta \Sigma^\dagger] \quad (15)$$

where the coupling $g({}^1P_1)$ is an unknown parameter, and could be determined by the data or could be calculated within QCD multipole expansion [25, 26]. Then, the amplitude for $X(1^{+-}, I=0) \rightarrow J/\psi\pi\pi$ is given by

$$\mathcal{M} \sim \epsilon_{ijk} \epsilon_X^i \epsilon_\psi^j (E_1 p_2^k + E_2 p_1^k), \quad (16)$$

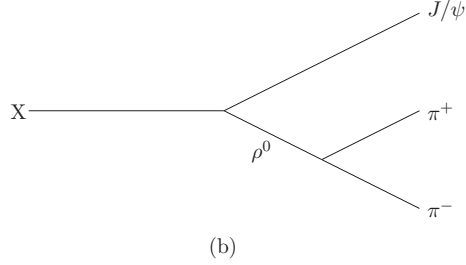
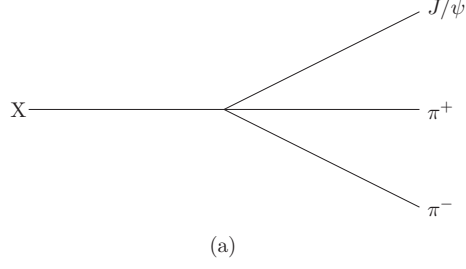
where E_1 and \vec{p}_1 are the energy and the three momentum of the pion 1, and similarly for the pion 2. As before, we have ignored the overall normalization, which is irrelevant to our discussion for the spectrum.

If X is a $D\overline{D}^*$ molecular state with $J^{PC} = 1^{+-}$, we cannot apply QCD multipole expansion. Still the chiral lagrangian approach will be applicable, and the above amplitude is still valid.

Using the above amplitude (16), it is straightforward to calculate the $\pi\pi$ invariant mass spectrum for $X(1^{+-}) \rightarrow \pi\pi J/\psi$. We show the result in Fig. 1 in the (green) dotted curve. Note that the dipion spectrum has a peak at low $m_{\pi\pi}$ region, if X has $J^{PC} = 1^{+-}$, which is disfavored by the preliminary Belle data [5]. Once high statistics data is obtained, one can easily check if $J^{PC}(X) = 1^{+-}$ is a correct assignment or not.

VI. $X(1^{++}, I=1) \rightarrow J/\psi\pi\pi$

This case includes that the decaying state is $I=1$ $D\overline{D}^*$ molecular state and the final dipion is in $I=1$, which would be dominated by ρ meson. Since $\rho^0 \rightarrow \pi^0\pi^0$ is forbidden by angular momentum conservation and Bose symmetry, the $\pi\pi$ in $X \rightarrow J/\psi\pi\pi$ should be charged pions in this case.



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FIG. 1: Feynman diagrams for $X(3872) \rightarrow J/\psi \pi \pi$

Since charge-conjugation symmetry C is conserved in strong interaction, we must have $C(\pi\pi) = -1$, if $C(X) = +$. Now let us construct a C -even chiral invariant operator. Since $I(X) = 1$ in this case, the X field transforms as $X_v \rightarrow U(x)X_v(x)U^\dagger(x)$ as discussed in Sec. II. Therefore the lowest order chiral lagrangian for $X(1^{++}, I = 1) \rightarrow J/\psi \pi \pi$ is given by

$$\mathcal{L} \sim \epsilon^{\mu\nu\alpha\beta} v_\mu \text{Tr}[(\rho_\nu - V_\nu)X_{v\alpha}] \psi_{v\beta}. \quad (17)$$

Then, the amplitude for $X(1^{++}, I = 1) \rightarrow J/\psi \pi \pi$ is given by

$$\mathcal{M} \sim \epsilon_{\mu\nu\alpha\beta} v^\mu p^\nu \epsilon_X^\alpha \epsilon_\psi^\beta \left[1 + \frac{m_\rho^2}{q^2 - m_\rho^2 + im_\rho \Gamma_\rho} \right], \quad (18)$$

where $q = p_1 + p_2$ and $p = p_1 - p_2$ as before, and $\Gamma_\rho = 153$ MeV is the total decay width of the ρ meson. The first term is from the contact interaction [Fig. 1 (a)], whereas the second term is due to the ρ exchange [Fig. 1 (b)].

In this case, the polarization of ρ and J/ψ tends to be perpendicular with each other, which can be tested by measuring the three-momentum of a lepton in $J/\psi \rightarrow l^+ l^-$ and the

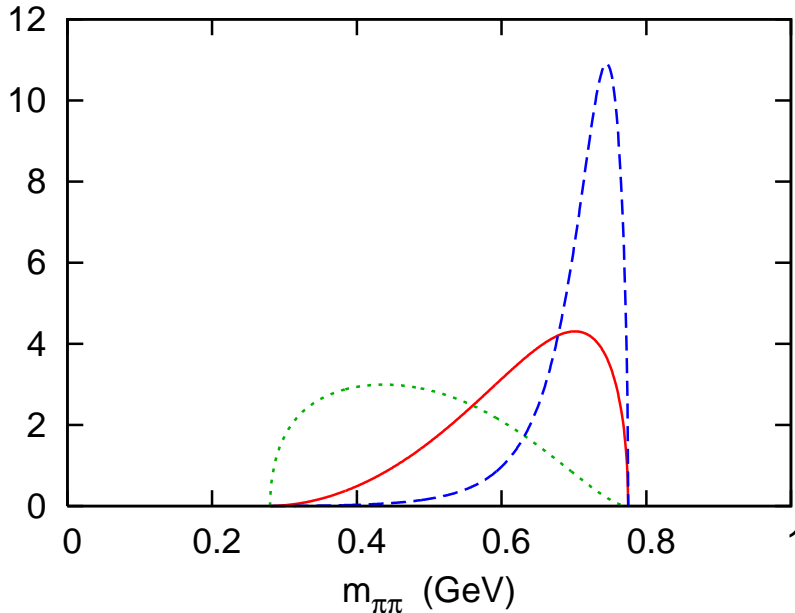


FIG. 2: Dipion invariant mass spectra for $J^{PC}(X) = 2^{--}$ or 3^{--} (red and solid), $J^{PC}(X) = 1^{+-}$ (green and dotted) and $J^{PC}(X) = 1^{++}$ (blue and dashed). The vertical scale is arbitrary for each case, and the areas under the curves is all the same.

three-momentum of a pion in $\rho \rightarrow \pi\pi$ decay.

VII. CONCLUSION

In this letter, we pointed out that the dipion invariant mass spectrum in $X(3872) \rightarrow J/\psi\pi\pi$ could be useful in determination of J^{PC} quantum number of the newly observed resonance $X(3872)$. In particular, the current preliminary data seems to already exclude the possibility $X = {}^1P_1$, since the dipion invariant mass spectrum has a peak at low $m_{\pi\pi}$ region in this case. This is apparently disfavored by the preliminary Belle data. On the other hand, if $X = {}^3D_2(2^{--})$ or $X = {}^3D_3(3^{--})$, then the $m_{\pi\pi}$ spectrum has a peak at high $m_{\pi\pi}$ region, which is consistent with the data. Also the interpretation in terms of $D\bar{D}^*$ molecular state predicts $m_{\pi\pi}$ spectrum to have a sharp peak near $m_{\pi\pi} \approx m_\rho$. This could be easily distinguishable in the future when more data are accumulated.

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